Thornton & Rex – Chapter 5

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X-Ray Scattering

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- X-Rays proven to be EM radiation by Max von Laue

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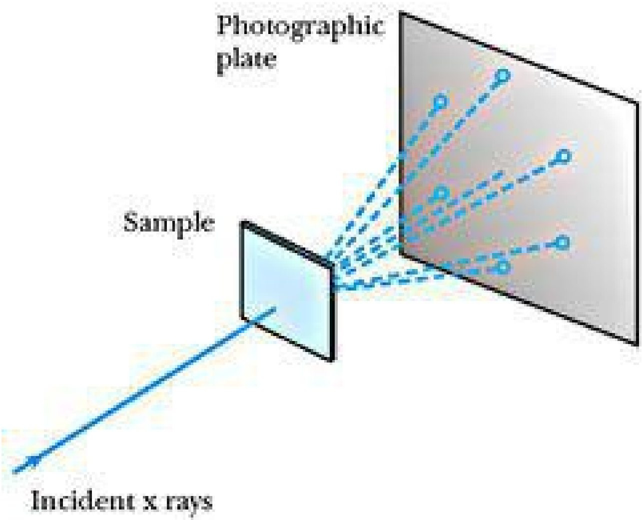
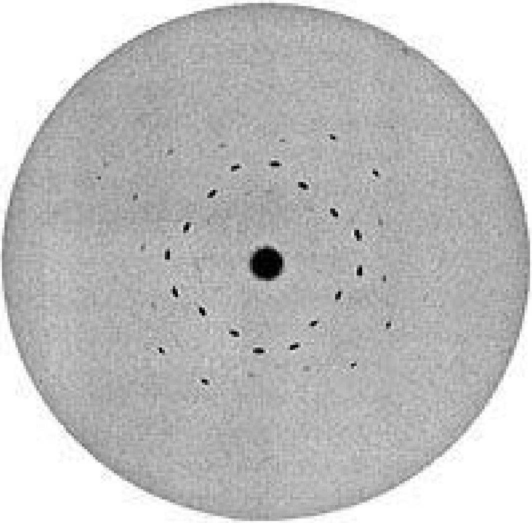
- Demonstrated both the wave nature of X-Rays and the lattice structure of crystals
- Won the Nobel Prize for Physics in 1914



(a)



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- The images surrounding the center could be interpreted as the reflection of the X-Ray beam on a unique set of lattice planes
  - Each image corresponds to a different set of planes
- They determined Bragg's law, which can be used to determine both wavelength and lattice spacing

$$n\lambda = 2d \sin\theta$$

De Broglie Waves

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Theory

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- The wavelength of a material particle, when converted into a photon of equal energy and momentum, will have a wavelength given by:

$$\lambda = \frac{h}{p}$$

Agreement with Bohr Model

- Imagine the electron in a Hydrogen atom as a standing wave orbiting the proton

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- Angular momentum given by $L = rp$

$$L = rp = \frac{nh}{2\pi} = n\hbar$$

Electron Scattering

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- Davison and Germer were awarded the Nobel Prize for Physics in 1937 for their discovery.

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- It seemed that the electrons acted like x rays as they diffracted in a new pattern based on the changed interatomic spacing within the material.

Wave Motion

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- One of these ways included looking at waves as a representation of matter

Theory

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- In their simplest form, these waves can be described by the equation

$$\Psi(x, t) = A \sin(kx - \omega t + \phi)$$

Principle of Superposition

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- For a continuous spectrum, the series is extended to a Fourier integral

$$\Psi(x, t) = \int \tilde{A}(k) \cos(kx - \omega t)$$

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- This is related to the uncertainty principle

Wave Equation

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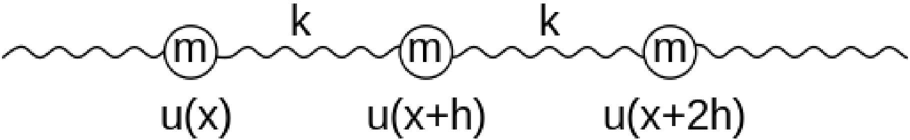
- In one dimension, this is simplified to

$$\frac{1}{v^2}\frac{\partial^2}{\partial t^2}\Psi(x,t) = \frac{\partial^2}{\partial x^2}\Psi(x,t)$$

Derivation of Wave Equation in 1D

- Imagine an array of masses m separated by displacement h by springs with spring constant k



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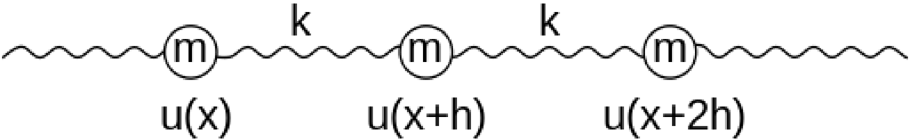
- $\Psi(x)$ represents the displacement from equilibrium of the mass at x



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- KL^2/M is the square of phase velocity, v

$$\frac{\partial^2}{\partial x^2}\Psi(x,t) = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\Psi(x,t)$$

Waves or Particles

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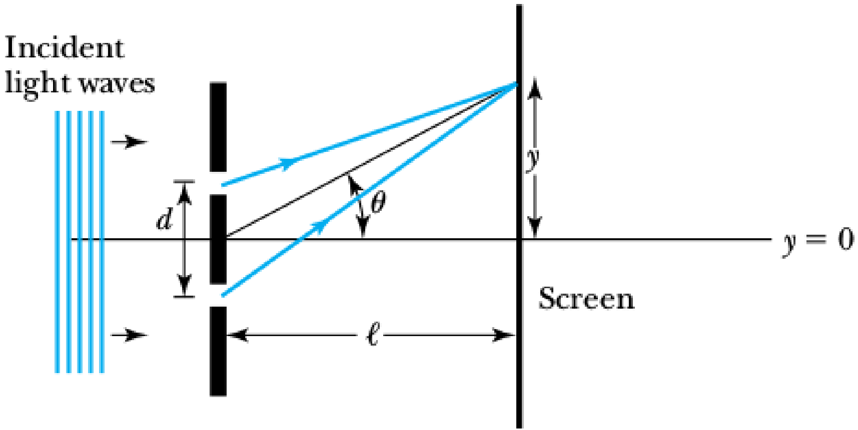
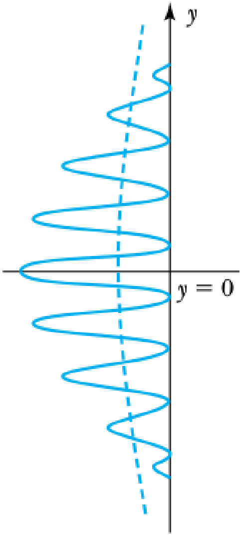
- This demonstrates the wave-nature of light


Incident
light waves
d
θ
y
y = 0
Screen
ℓ

:



y
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- If the intensity of the light is reduced, we observe discrete flashes of light on the screen
- Over time, the sum of these flashes recreates the interference pattern

(a) 20 counts

(b) 100 counts

(c) 500 counts

(d) ~4000 counts

Simulation and photos courtesy of Julian V. Noble.

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- The same nature was observed for electrons as was observed for light

Uncertainty Principle

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- In the case of a Gaussian wave packet

$$\Delta k \Delta x = \frac{\Delta p}{\hbar} \Delta x = \frac{1}{2} \implies \Delta p \Delta x = \frac{\hbar}{2}$$

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- Time-energy form

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

Probability, Wave Functions, and the Copenhagen Interpretation

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- In electrodynamics, $\mathbf{E}$ or $\mathbf{B}$ serves as the wave function
- For matter waves, Ψ determines the probability of finding a particle at a particular location in space at a time t

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- If Ψ is complex, $\Psi^* \Psi$ is used when finding probabilities
- When only interested in a single dimension y at a given time t, $P(y) = \Psi^* \Psi = |\Psi|^2$ is the probability of observing a particle in the interval between y and $y + \mathrm{d}\, y$

Normalization

- The probability of finding a particle *somewhere* must be unity, and so the probability density is integrated over all space

$$\int_{-\infty}^{\infty} P(y)\,\mathrm{d}\,y = \int_{-\infty}^{\infty} |\Psi(y,t)|^2\,\mathrm{d}\,y = 1$$

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  - Einstein stated "God does not throw dice" in defiance of the interpretation

Particle in a Box

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- The possible values of λ are quantized as a result, giving discrete energy levels of the particle. This explains the Bohr model

Thank you for listening!