Principal Component Analysis of Cepheid Variable Stars

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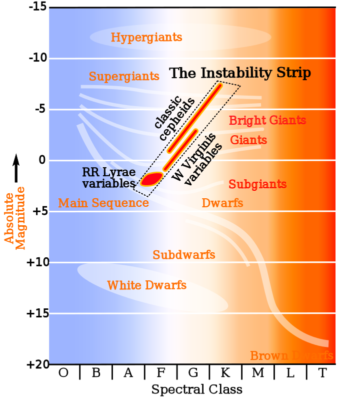
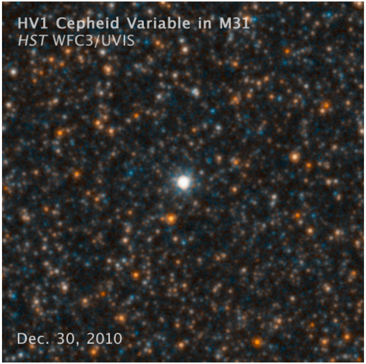
Variable Stars



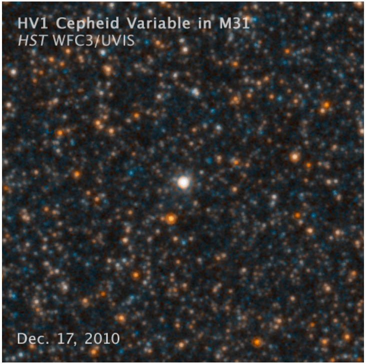
- stars whose size and luminosity fluctuate
- mostly giant stars
- instability strip

Classical Cepheid Variable Stars



- population I (high metallicity)
- 4–20 solar masses
- up to 100000 solar luminosities
- pulsation period can range from days to months
- period-luminosity relationship

Classical Cepheid Variable Stars



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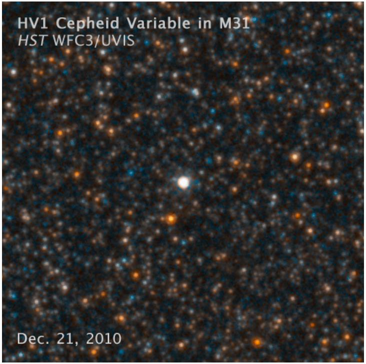

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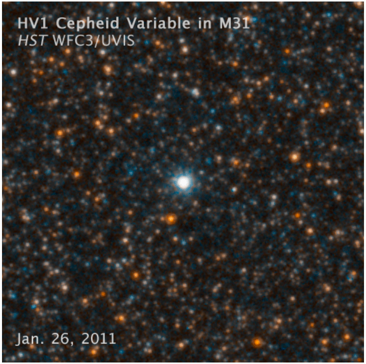

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Lightcurves

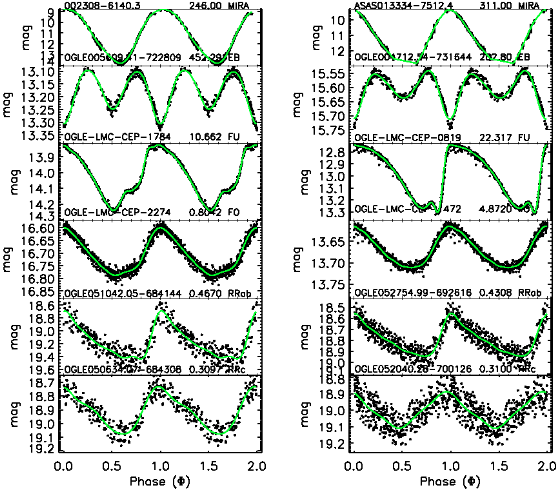
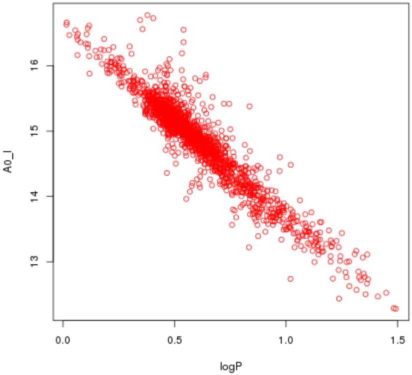


Figure: Lightcurves of different classes of variable stars

Cepheid Period-Luminosity Relationship



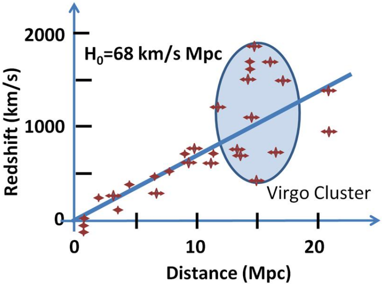
$$\underbrace{A_0}_{\text{mean magnitude}} = a \log \underbrace{P}_{\text{period}} + c$$

- a Cepheid's period of oscillation is related to its mean luminosity
- approximate a linear model which gives a and c
- this makes A_0 a function of $\log P$ and some fixed constants

$$\underbrace{m}_{\text{apparent magnitude}} - \underbrace{M}_{\text{absolute magnitude}} = 5 \log \left(\frac{d}{10} \right) - 5$$

$$\underbrace{d}_{\text{distance in parsecs}} = 10^{\frac{m-M}{5}+1}$$

Hubble's Law



$$\underbrace{v}_{\text{redshift velocity}} = \underbrace{H_0}_{\text{Hubble's constant}} \underbrace{d}_{\text{distance}}$$

- Hubble's law describes the velocity of the expansion of the Universe
- redshift measurements give us v
- Cosmic Microwave Background (CMB) only gives us a measure of $H_0^2 \Omega$
- independent measure of H_0 is needed to find density of Universe, Ω

Fourier Analysis of Lightcurves

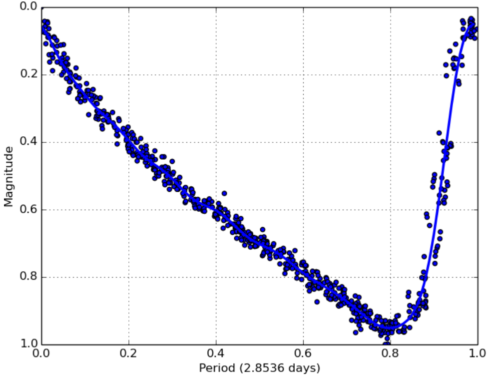


Figure: Fundamental Mode Cepheid in the LMC with 7^{th}order Fourier fit from OGLEIII

- assume basis lightcurve to be sinusoidal
- find values of best fit for A_0, A_k and Φ_k
- for n^{th}order fit, requires $2n+1$ parameters

$$\underbrace{A(t)}_{\text{mag at time } t} = \underbrace{A_0}_{\text{mean mag}} + \sum_{k=1}^{n} \underbrace{A_k}_{\substack{\text{scaling} \\ \updownarrow}} \sin(\underbrace{k}_{\substack{\text{scaling} \\ \leftrightarrow}} \omega t + \underbrace{\Phi_k}_{\substack{\text{shift} \\ \leftrightarrow}})$$

Fourier Parameters versus $\log P$

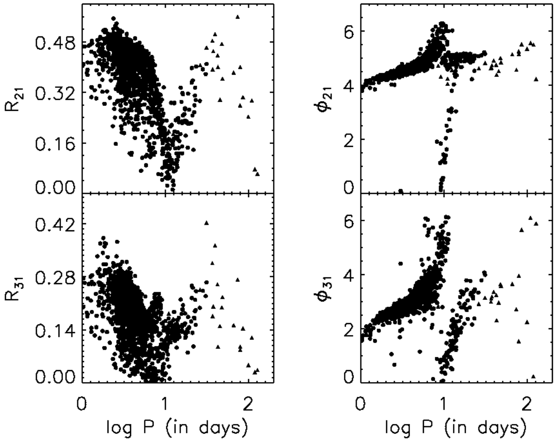
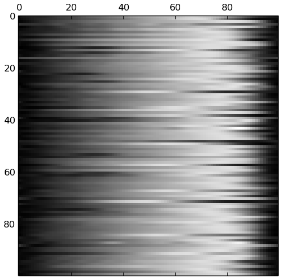


Figure: Fourier parameter ratios of 1829 fundamental mode Cepheids in LMC

Principal Component Analysis of Lightcurves

- data decides the basis lightcurves
- construct a matrix of all the stars' lightcurves stacked vertically
- find the covariance matrix of this matrix $(\mathbf{A}^\top \mathbf{A})$
- eigenvectors $(\mathbf{EV})$ of the covariance matrix are the basis lightcurves
- scalar coefficents are the principle scores (PC)
- n^{th}order fit requries only n parameters for each star, in addition to the n eigenvectors for the whole dataset



$$PC_i = \mathbf{A} \cdot \mathbf{EV}_i$$

$$\mathbf{A} = \sum_{i=1}^{n} PC_i \mathbf{EV}_i$$

Figure: First 100 rows of input matrix

Principal Component Analysis of Lightcurves

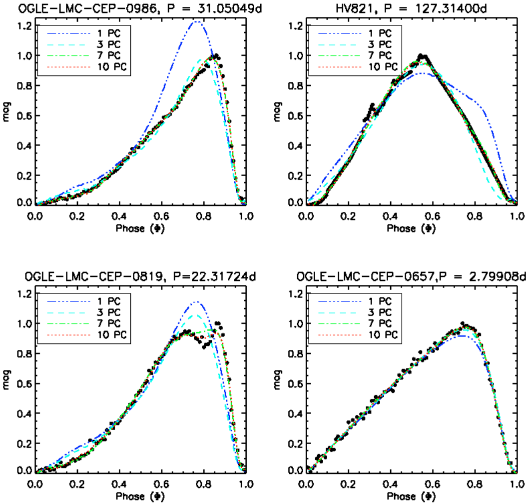


Figure: Cepheids with varying order PCA fits

Principal Scores versus $\log P$

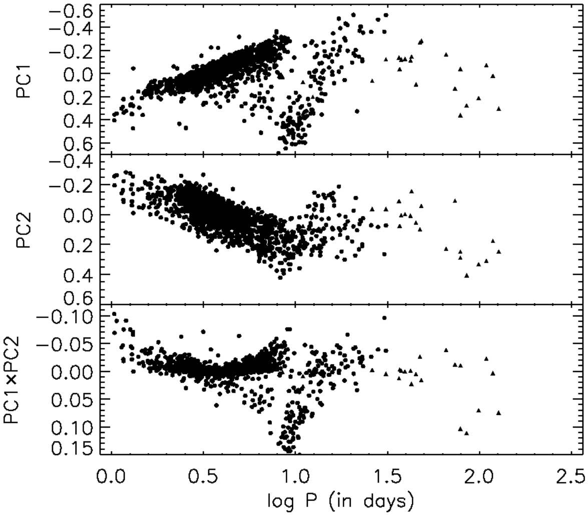
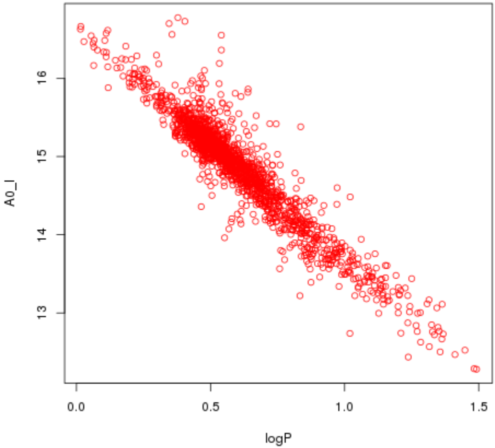


Figure: Principal scores 1 and 2 as functions of $\log P$ for 1829 fundamental mode Cepheids in LMC

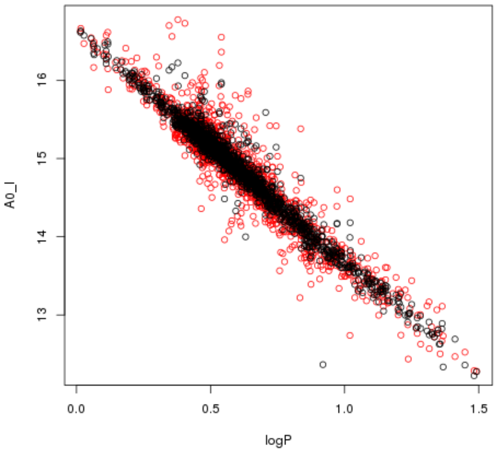
Cepheid Period Luminosity Relationship

- $A_0 = a \log P + c$



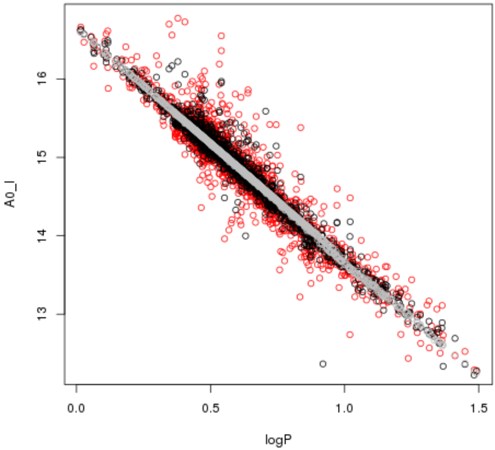
Cepheid Period Luminosity Color Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(I - V) + c$



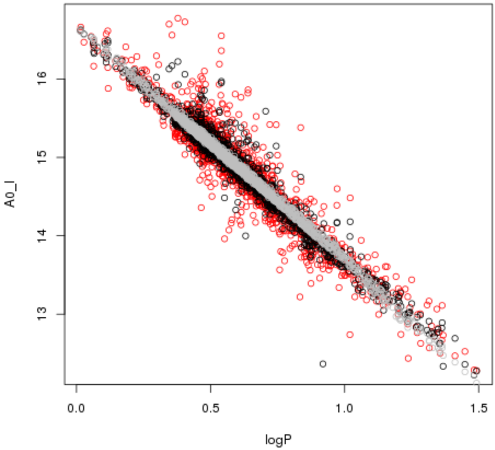
Cepheid Period Luminosity Principal Component Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(I - V) + c$
- $A_0 = a \log P + bPC_1 + c$



Cepheid Period Luminosity Principal Component Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(I - V) + c$
- $A_0 = a \log P + bPC_2 + c$



Period Luminosity Principal Component Relationship

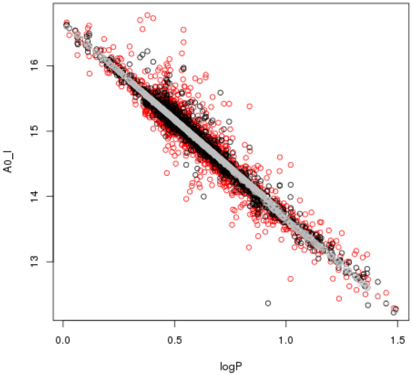


Figure: A_0 fitted with PC_1 vs $\log P$

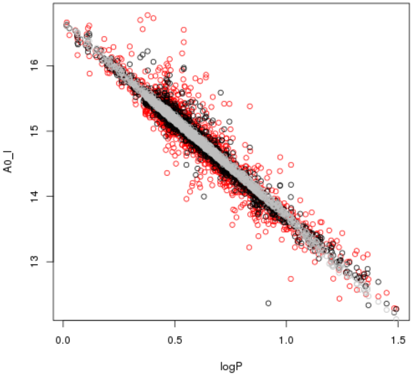


Figure: A_0 fitted with PC_2 vs $\log P$

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References