Principal Component Analysis of Cepheid Variable Stars

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Variable Stars

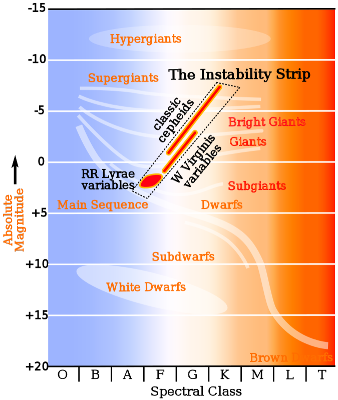

Figure: Hertzsprung–Russell diagram

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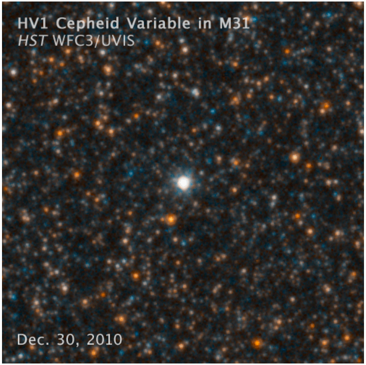
Classical Cepheid Variable Stars



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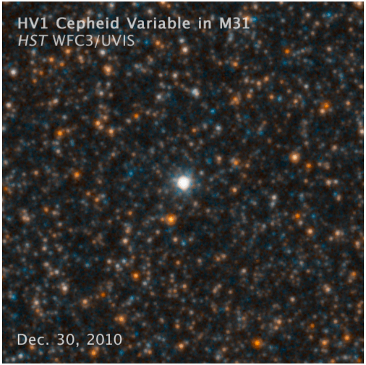

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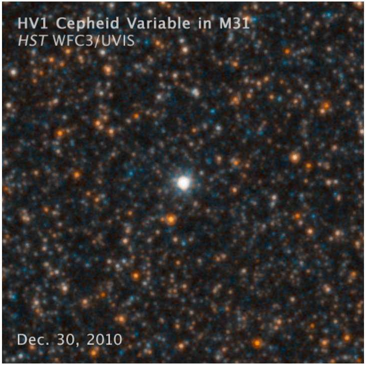

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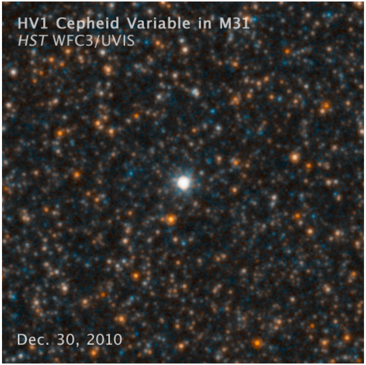
Classical Cepheid Variable Stars



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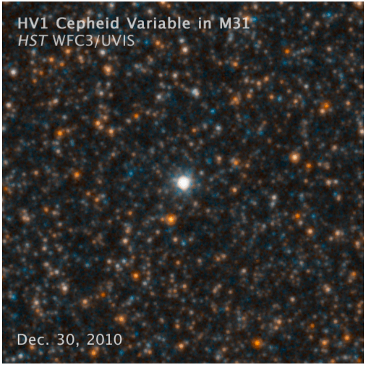

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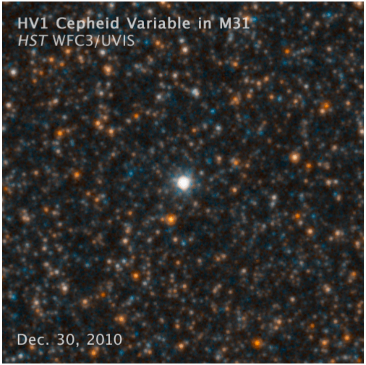

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- period-luminosity relationship

Lightcurves

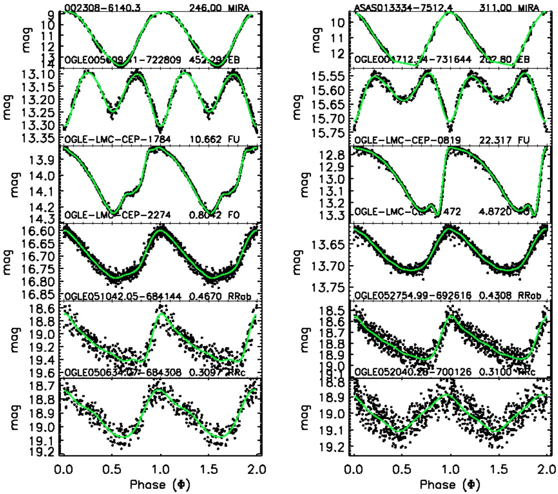
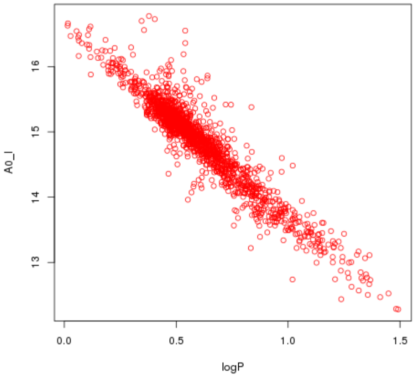


Figure: Lightcurves of different classes of variable stars

Cepheid Period-Luminosity Relationship



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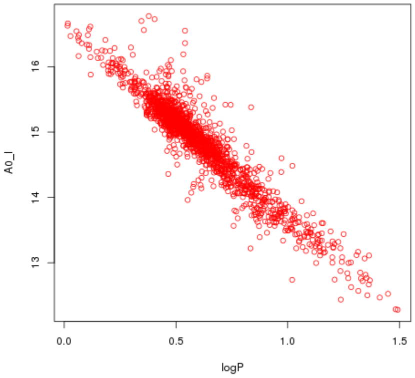
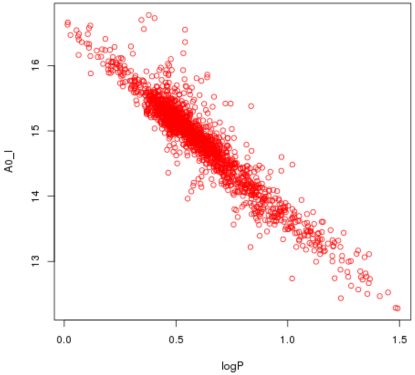


- a Cepheid's period of oscillation is logarithmically related to its mean luminosity



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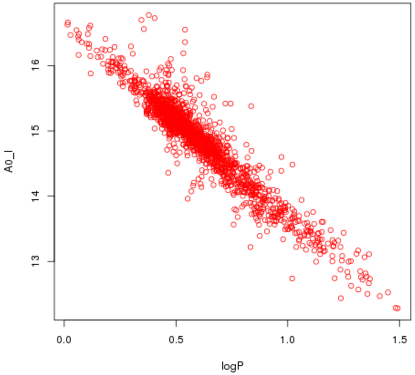

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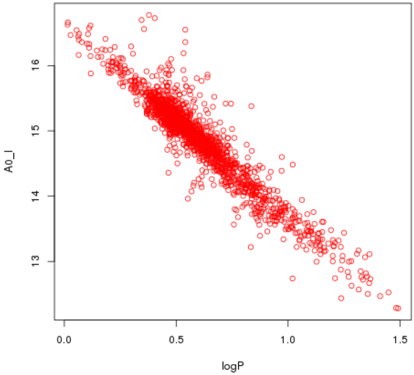

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$$\underbrace{m}_{\text{apparent magnitude}} - \underbrace{M}_{\text{absolute magnitude}} = 5 \log \left(\frac{d}{10} \right) - 5$$

$$\underbrace{d}_{\text{distance in parsecs}} = 10^{\frac{m-M}{5}+1}$$

Cosmic Microwave Background

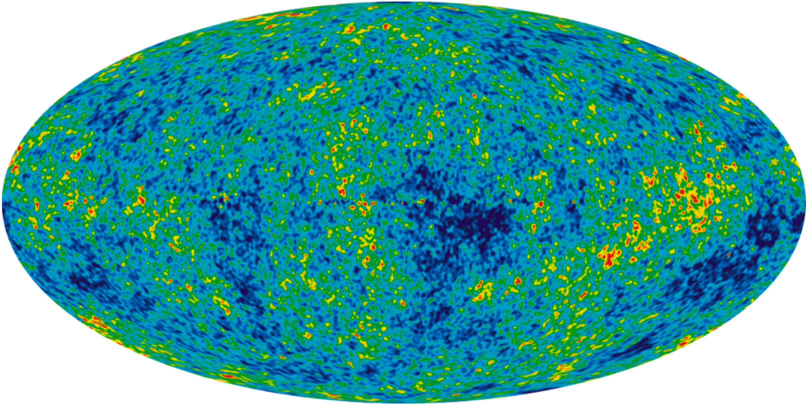
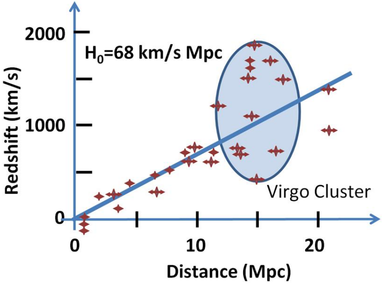


Figure: Full sky map of CMB, made from 9 years of WMAP data.

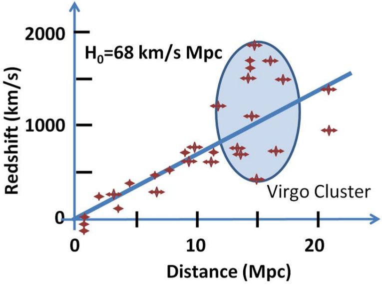


$$\underbrace{v}_{\text{redshift velocity}} = \underbrace{H_0}_{\text{Hubble's constant}} \underbrace{d}_{\text{distance}}$$

$$H_0 = \frac{v}{d}$$

- Hubble's law describes the velocity of the expansion of the Universe

Hubble's Law



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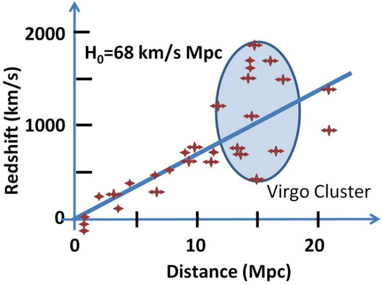

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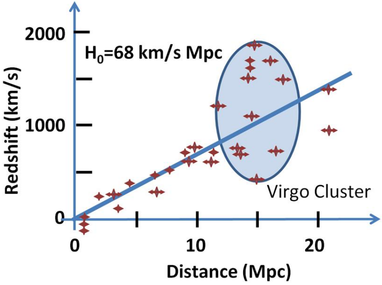

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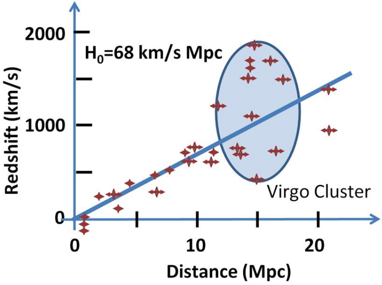

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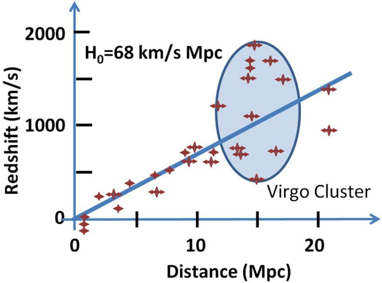

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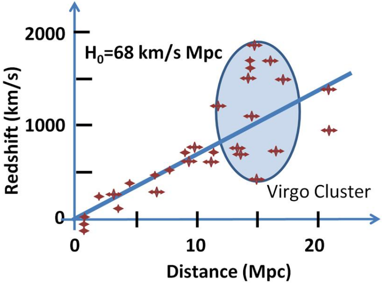

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- Ω is the average density of the Universe
- independent measure of H_0 is needed to find Ω

Fourier Analysis of Lightcurves

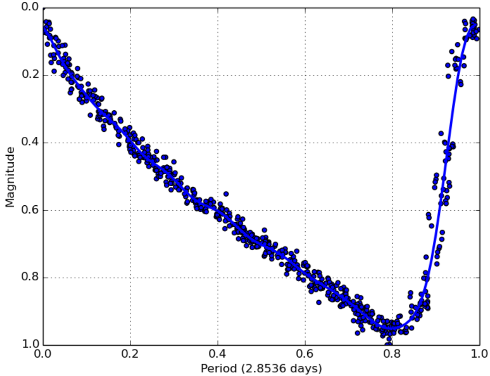
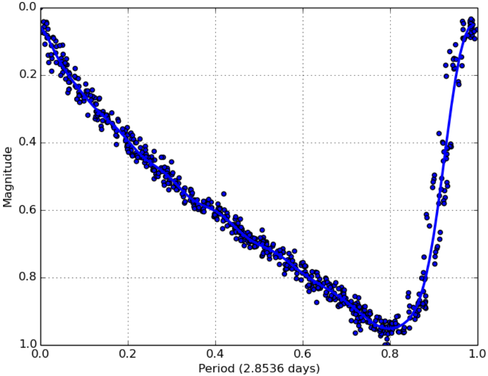


Figure: Fundamental Mode Cepheid in the LMC with 7thorder Fourier fit from OGLEIII

- assume basis lightcurve to be sinusoidal

$$\underbrace{A(t)}_{\text{mag at time } t} = \underbrace{A_0}_{\text{mean mag}} + \sum_{k=1}^{n} \underbrace{A_k}_{\text{scaling } \updownarrow} \sin(\underbrace{k}_{\text{scaling } \leftrightarrow} \omega t + \underbrace{\Phi_k}_{\text{shift } \leftrightarrow})$$



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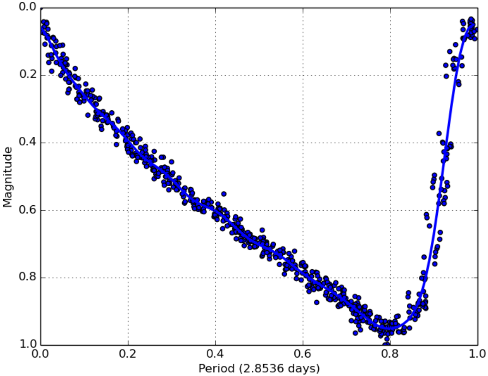

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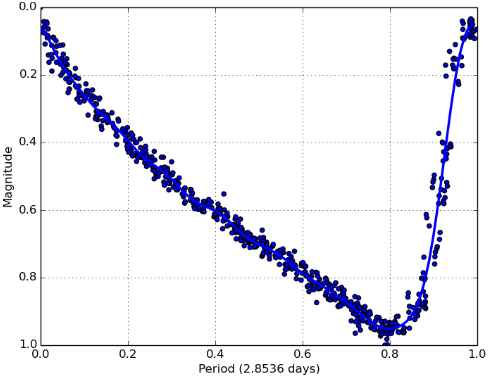

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- for n^{th}order fit, requires $2n + 1$ parameters

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Fourier Parameters versus $\log P$

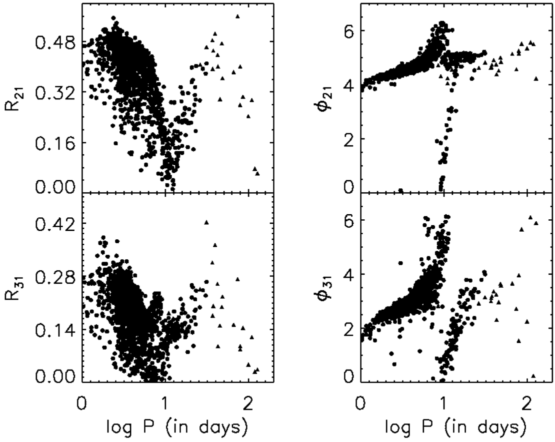

Figure: Fourier parameter ratios of 1829 fundamental mode Cepheids in LMC

Principal Component Analysis of Lightcurves

- data decides the basis lightcurves

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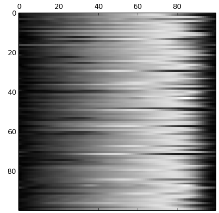

Figure: First 100 rows of input matrix $\mathbf{A}$

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- construct a matrix of all the stars' lightcurves stacked vertically ($\mathbf{A}$)

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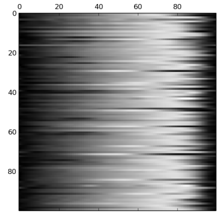

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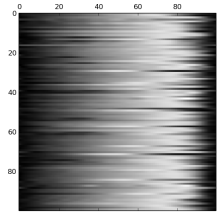

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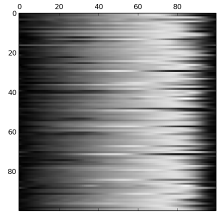

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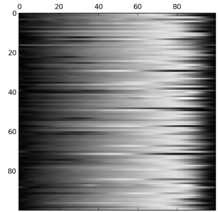

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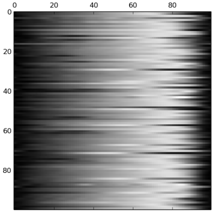
- scalar coefficents are the principle scores (PC)

Principal Component Analysis of Lightcurves



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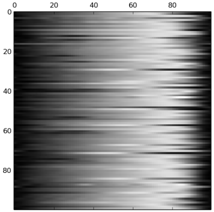

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- eigenvectors $(\mathbf{EV})$ of the covariance matrix are the basis lightcurves
- scalar coefficents are the principle scores (PC)
- if all eigenvectors are used, a star's lightcurve can be reconstructed perfectly
- using only the most significant eigenvectors gives a very good approximation of the original lightcurve

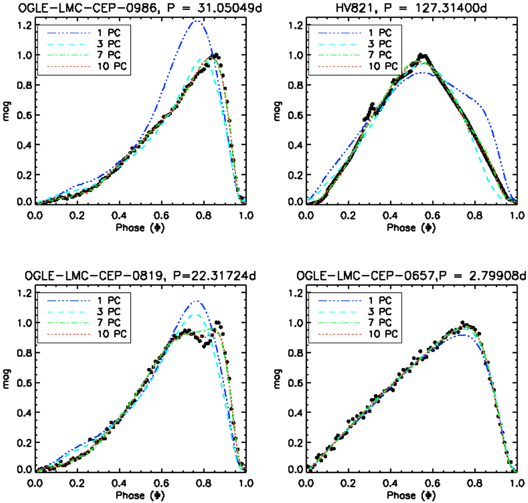


Figure: Cepheids with varying order PCA fits

Principal Scores versus $\log P$

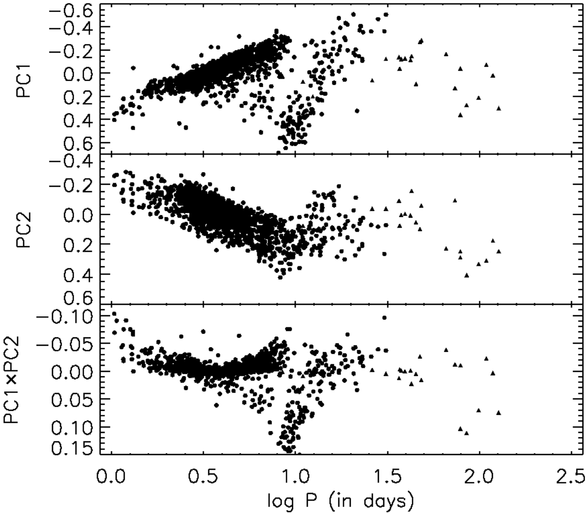
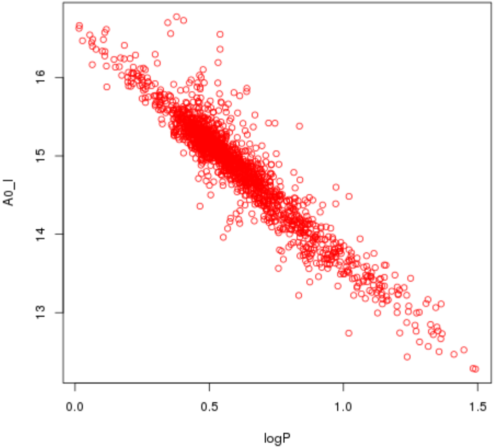


Figure: Principal scores 1 and 2 as functions of $\log P$ for 1829 fundamental mode Cepheids in LMC

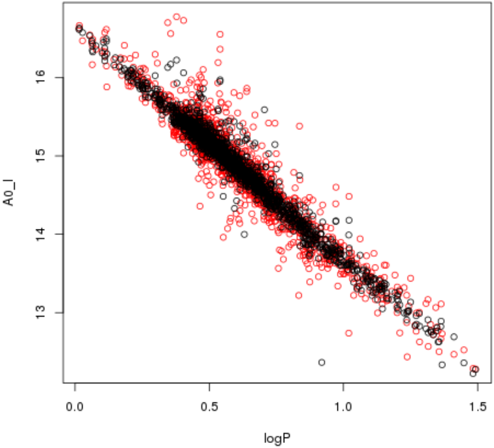
Cepheid Period Luminosity Relationship

- $A_0 = a \log P + c$



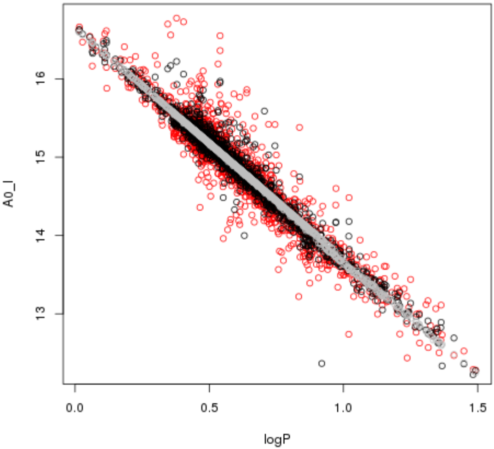
Cepheid Period Luminosity Color Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(I - V) + c$



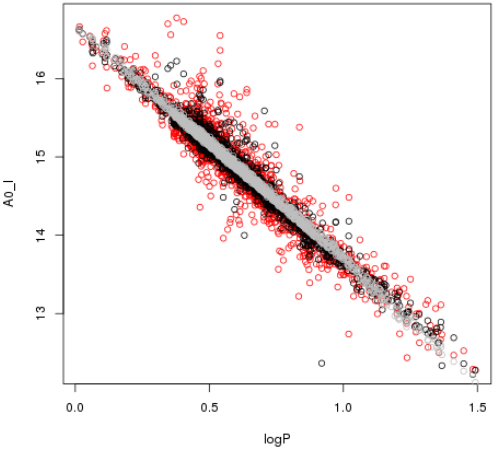
Cepheid Period Luminosity Principal Component Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(I - V) + c$
- $A_0 = a \log P + bPC_1 + c$



Cepheid Period Luminosity Principal Component Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(I - V) + c$
- $A_0 = a \log P + bPC_2 + c$



Period Luminosity Principal Component Relationship

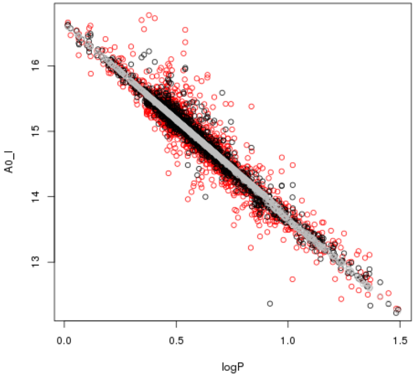


Figure: A_0 fitted with PC_1 vs $\log P$

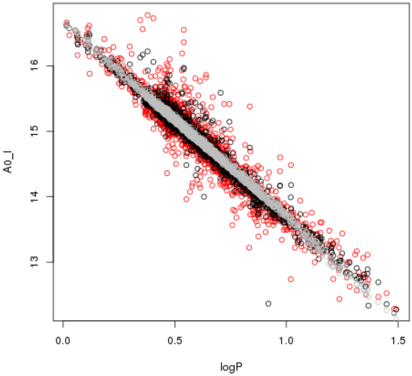


Figure: A_0 fitted with PC_2 vs $\log P$

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