Principal Component Analysis of Cepheid Variable Stars

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NYSSAPS 110th topical symposium

Variable Stars

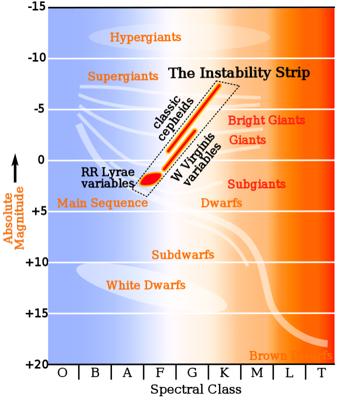

Figure: Hertzsprung–Russell diagram

- stars whose brightness varies over time

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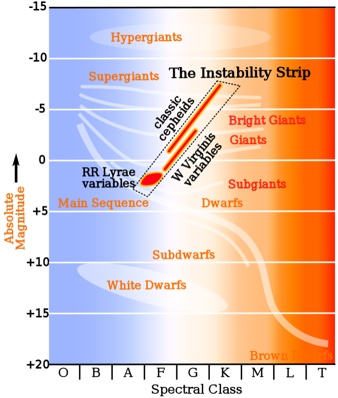

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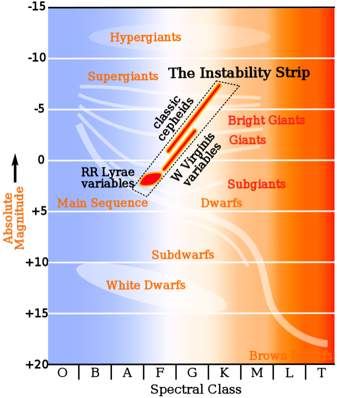

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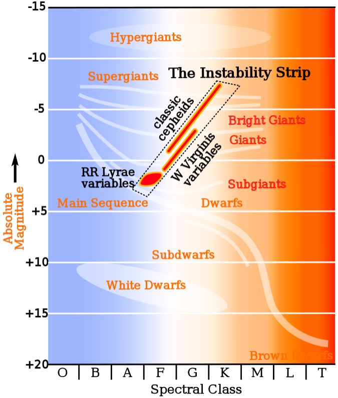

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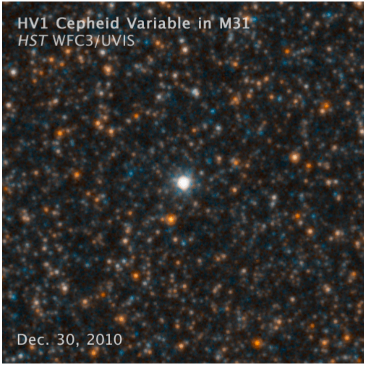
- some are only appear to be variable, while others are intrinsicly variable
- intrinsic variables are mostly giant stars
- occur in the instability strip

Classical Cepheid Variable Stars



- intrinsically variable

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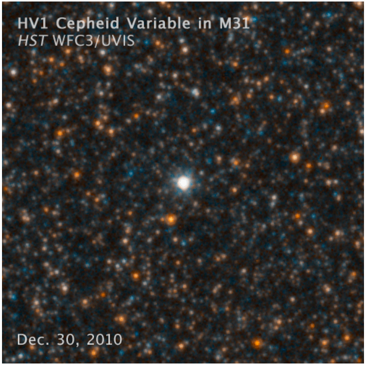

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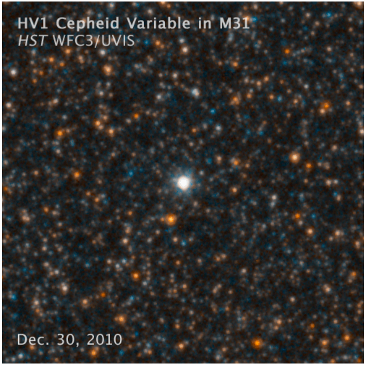

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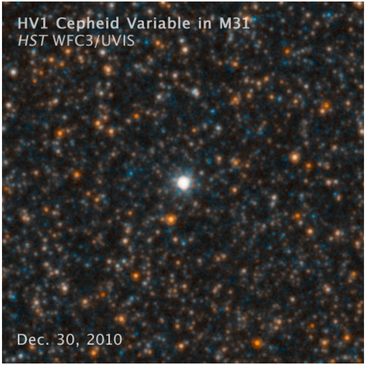

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- pulsation follows a period-luminosity relationship

Lightcurves

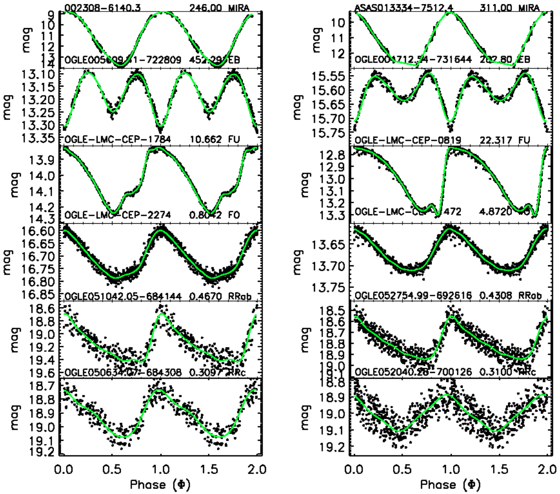
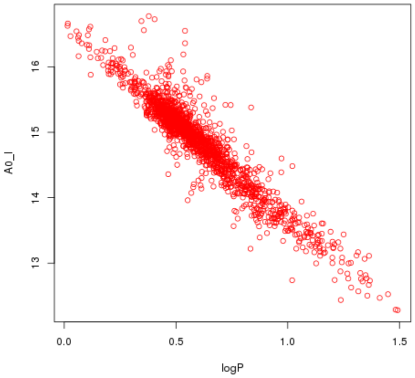


Figure: Lightcurves of different classes of variable stars

Cepheid Period–Luminosity Relationship



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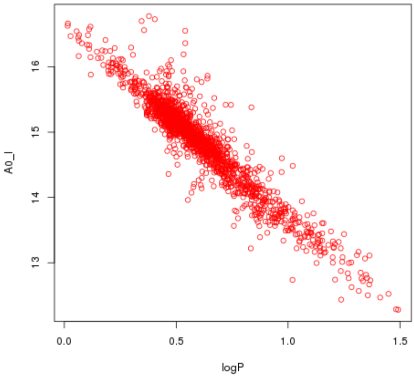

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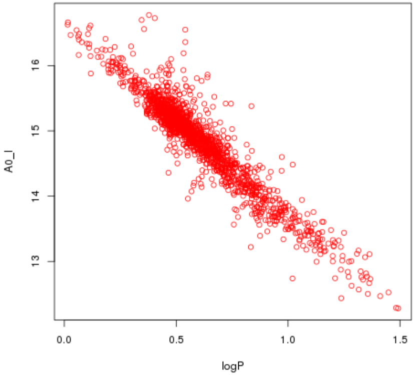

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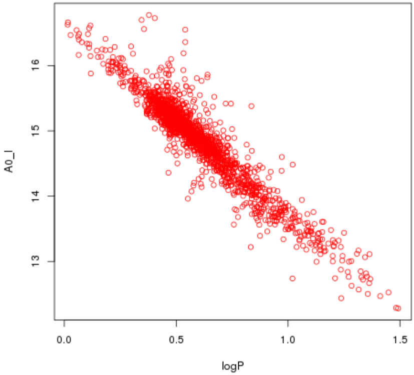

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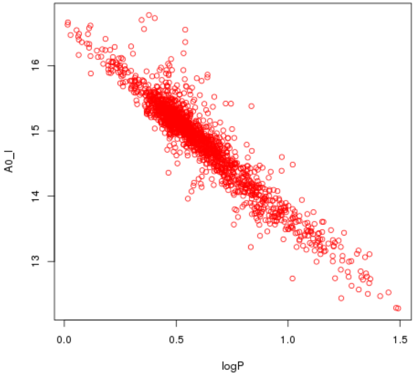

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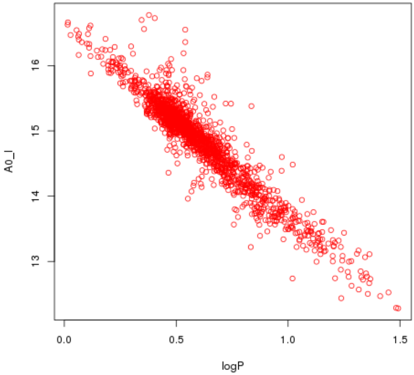

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Cepheids as a Standard Candle

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- interstellar matter absorbs some of the light, causing the apparent magnitude to decrease further

$$\underbrace{m}_{\text{apparent magnitude}} - \underbrace{M}_{\text{absolute magnitude}} = 5\log\left(\frac{d}{10}\right) - 5 \implies \underbrace{d}_{\text{distance in parsecs}} = 10^{\frac{m-M}{5}+1}$$

Cosmic Microwave Background

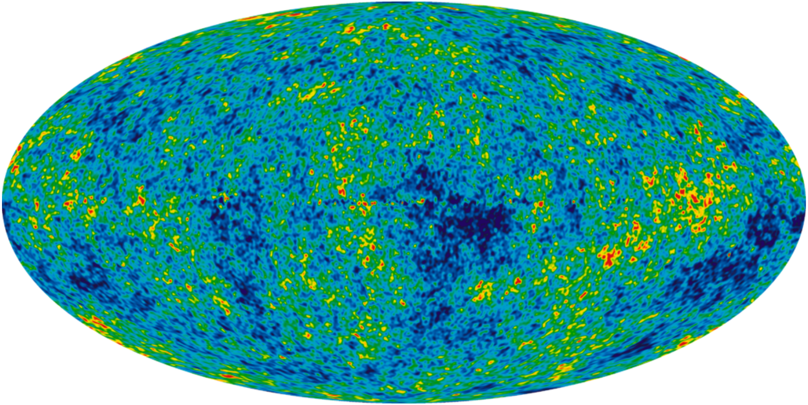
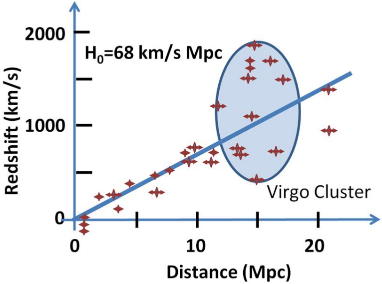


Figure: Full sky map of CMB, made from 9 years of WMAP data.

Hubble's Law



$$\underbrace{v}_{\text{redshift velocity}} = \underbrace{H_0}_{\text{Hubble's constant}} \underbrace{d}_{\text{distance}}$$

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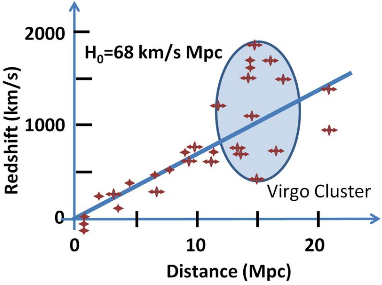

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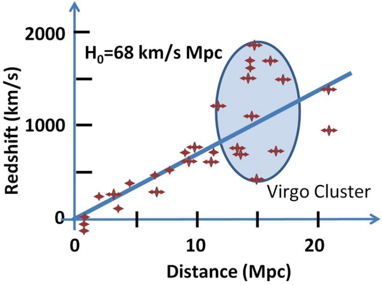

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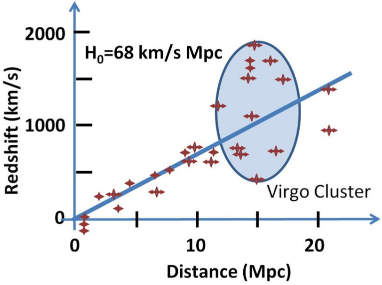

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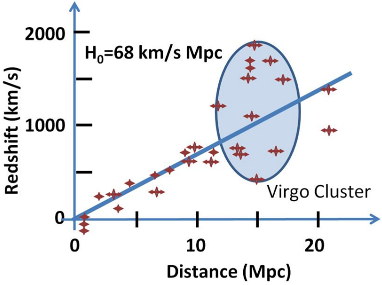

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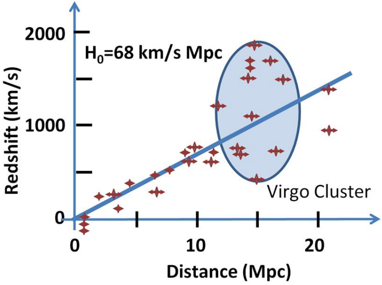

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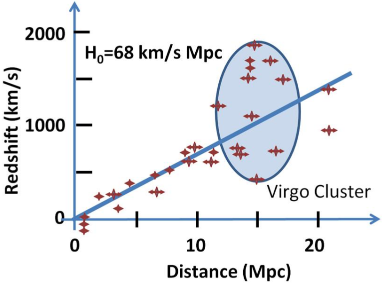

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- independent measure of H_0 is needed to find Ω

Fourier Analysis of Lightcurves

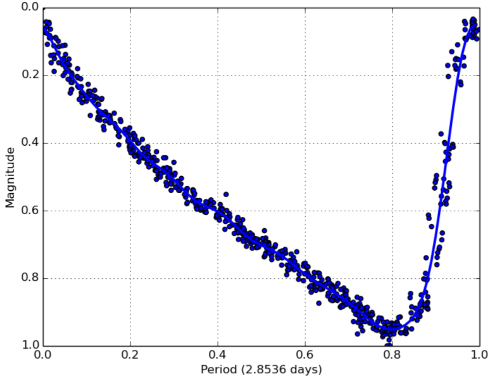

Figure: Fundamental Mode Cepheid in the LMC with 7th order Fourier fit from OGLEIII

- assume basis lightcurve to be sinusoidal

$$\underbrace{A(t)}_{\text{mag at time } t} = \underbrace{A_0}_{\text{mean mag}} + \sum_{k=1}^{n} \underbrace{A_k}_{\substack{\text{scaling} \\ \updownarrow}} \sin(\underbrace{k}_{\substack{\text{scaling} \\ \leftrightarrow}} \omega t + \underbrace{\Phi_k}_{\substack{\text{shift} \\ \leftrightarrow}})$$

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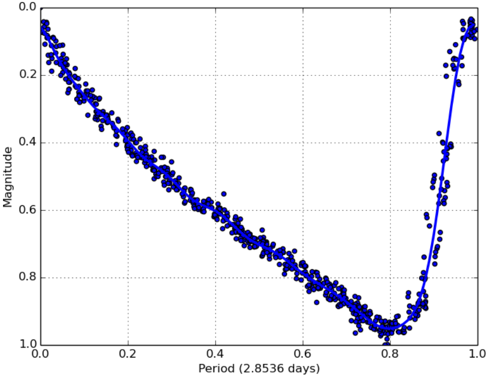

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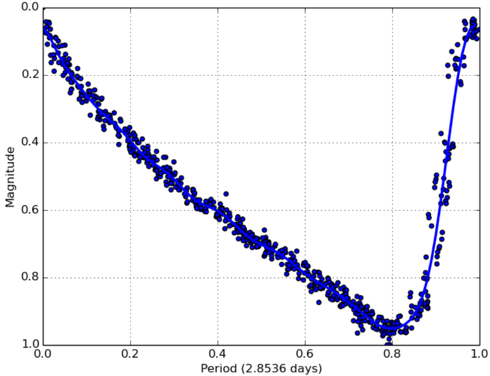

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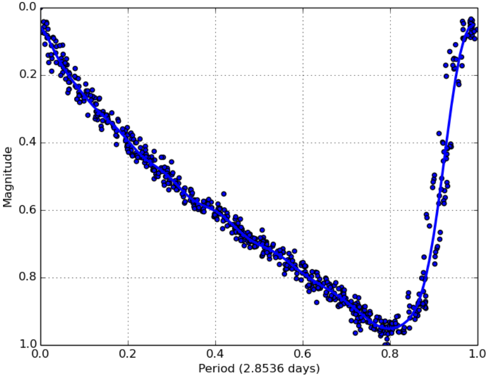

Figure: Fundamental Mode Cepheid in the LMC with 7thorder Fourier fit from OGLEIII

- assume basis lightcurve to be sinusoidal
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- for n^{th}order fit, requires $2n+1$ parameters

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Fourier Parameters versus $\log P$

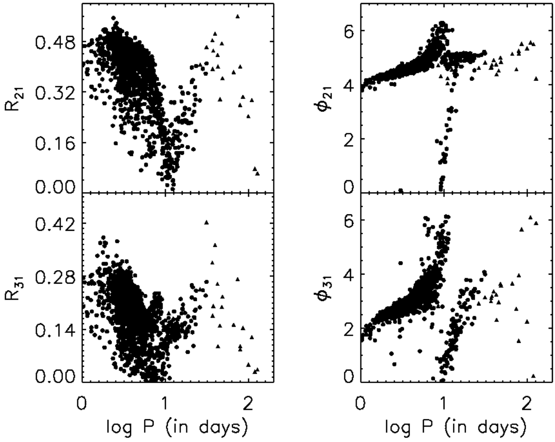

Figure: Fourier parameter ratios of 1829 fundamental mode Cepheids in LMC

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- any given star's lightcurve can be recreated by a linear combination of the eigenvectors, with a scalar coefficient (PC) called a "principle score"

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- dropping all but the most significant eigengectors gives a very close approximation to the original data with fewer parameters

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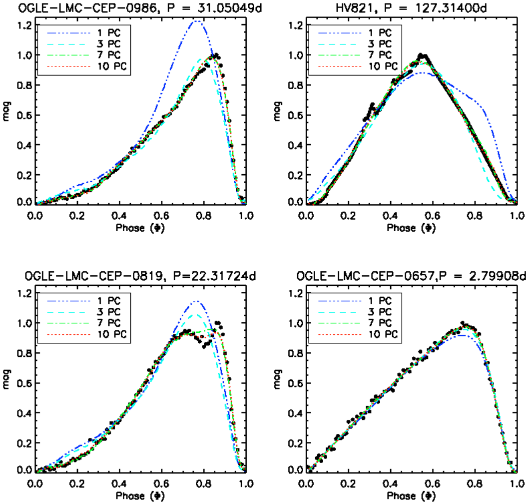


Figure: Cepheids with varying order PCA fits

Principal Scores versus $\log P$

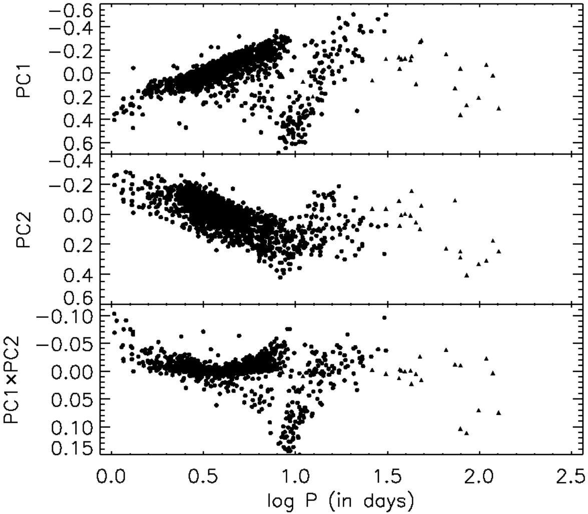
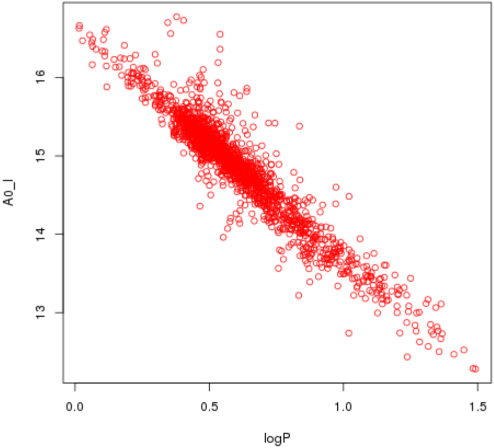


Figure: Principal scores 1 and 2 as functions of $\log P$ for 1829 fundamental mode Cepheids in LMC

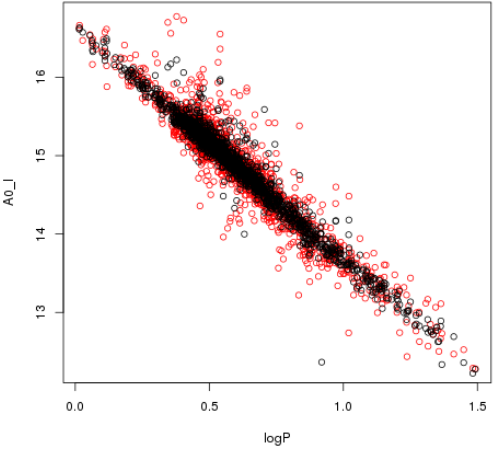
Cepheid Period Luminosity Relationship

- $A_0 = a \log P + c$



Cepheid Period Luminosity Color Relationship

- $A_0 = a \log P + c$
- $A_0 = a \log P + b(B - V) + c$



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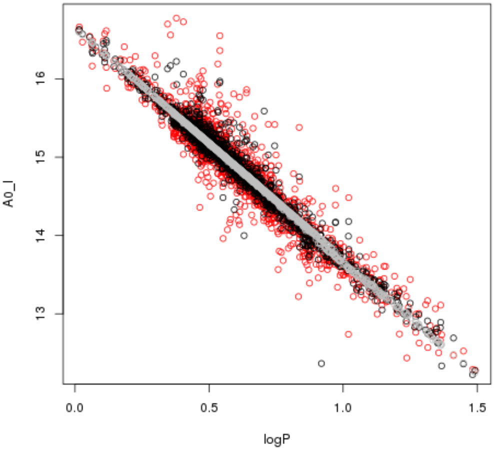

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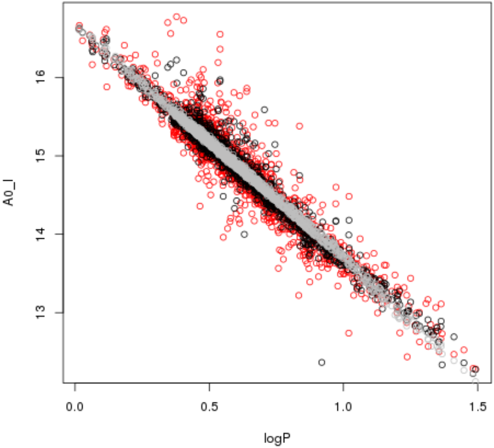

Period Luminosity Principal Component Relationship

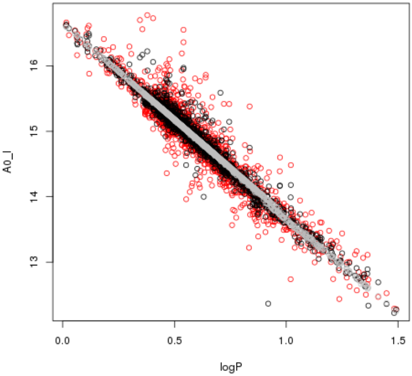

Figure: A_0 fitted with PC_1 vs $\log P$

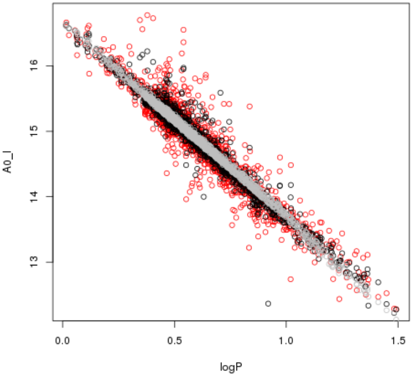

Figure: A_0 fitted with PC_2 vs $\log P$

Conclusions

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- principal scores are independent of interstellar reddening
- more precise distance measurements can be used to better calculate Planck's constant (H_0)
- combining a precise measurement of Planck's constant with the measurements obtained from the CMB ($H_0^2\Omega$) can be used to find a precise measurement of the density of the Universe (Ω)

References

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